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Analysis of non-Markovian Coupling of a Lattice-Trapped Atom to Free Space*

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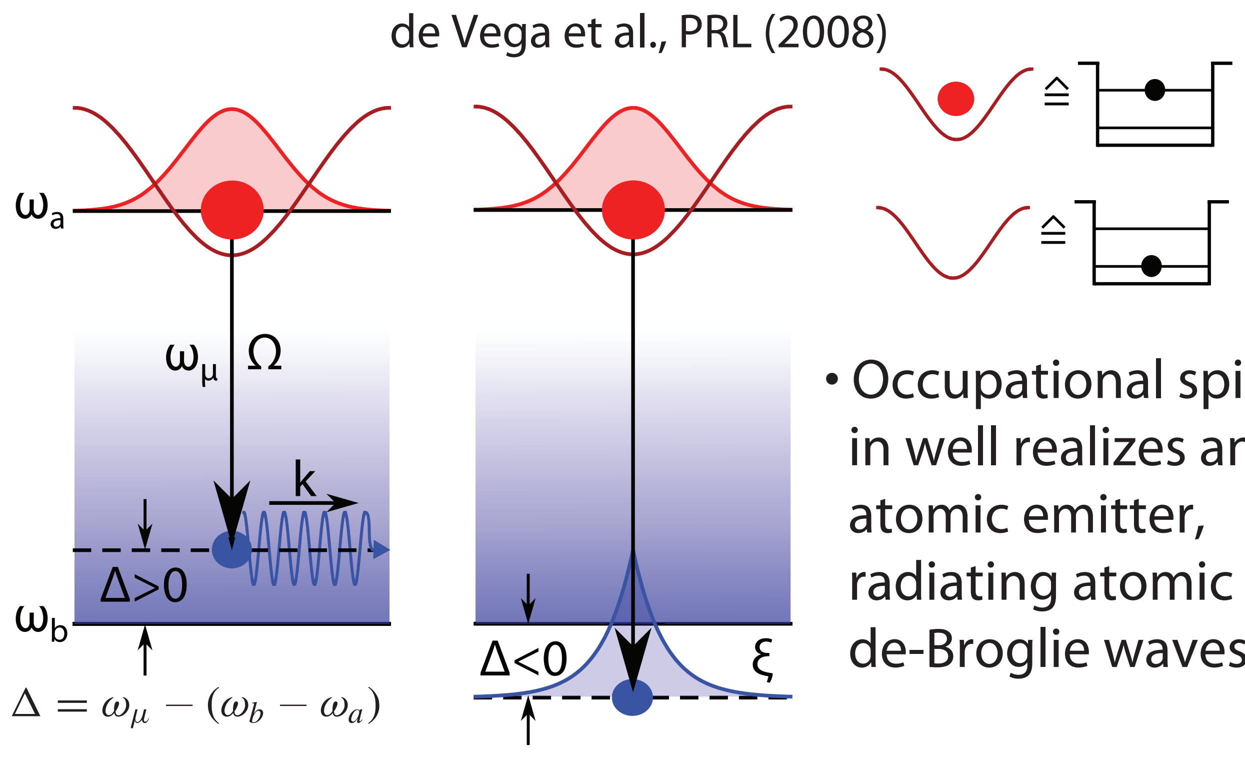
Weisskopf-Wigner Hamiltonian

de Vega et al., PRL (2008)



- Occupational spin in well realizes an atomic emitter, radiating atomic de-Broglie waves.

$$\hat{H}_{ab} = \sum_{\vec{k}} \frac{\hbar\Omega}{2}\gamma_{\vec{k}} e^{-i\Delta_k t}\hat{b}^\dagger_{\vec{k}}|0_a\rangle\langle 1_a| + \mathrm{H.c.}$$

$\gamma_k = \sqrt{\frac{2\pi^{1/2}a_{ho}}{L}}\exp\left[-\frac{1}{2}k^2a_{ho}^2\right]$ 1D Franck Condon Factor

$\Delta_k = \frac{\hbar k^2}{2m} - \Delta$ Free particle dispersion

- Single excited emitter coupled to free particle modes
- Analogous to atom-light case with different dispersion relation - diverging density of states at edge!
- Divergence causes non-Markovian behavior over large range of experimentally relevant parameters

1D Solution of the Model

Solution Ansatz: Single atomic emitter excited or else one matter wave present in system

$$|\Psi(t)\rangle = A(t)|1_a,\{0\}_{\vec{k}}\rangle + \sum_{\vec{k}} B_{\vec{k}}(t)|0_a,1_{\vec{k}}\rangle$$

Schrödinger equation gives for state amplitudes

$$\dot{A}(t) = i\sum_k g^*_{\vec{k}} e^{-i\Delta_k t}B_{\vec{k}}(t),\ \dot{B}_{\vec{k}}(t) = i g_{\vec{k}} e^{i\Delta_k t}A(t)$$

Integration and formal substitution leads to

$$\dot{A}(t) = -\int_0^t dt' A(t')G_{1D}(t-t') \qquad G_{1D}(\tau) = \frac{(\Omega/2)^2}{\sqrt{1+i\omega_0\tau/2}}e^{i\Delta\tau}$$

Bath correlation function

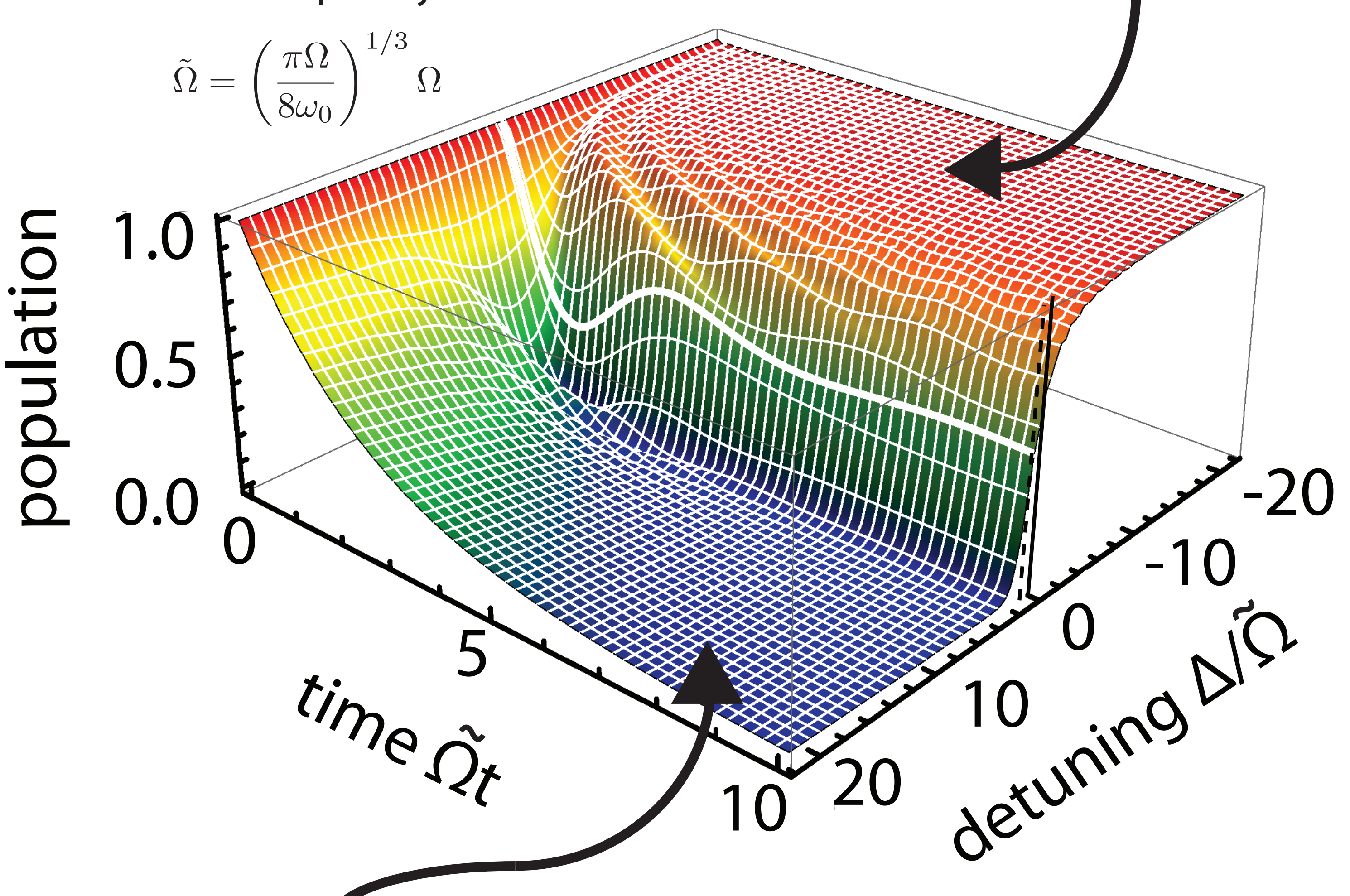
Solve by Laplace transform

$$\tilde{A}(s) = \frac{1}{s+\tilde{G}_{1D}(s)}$$

Schematically: $A(t)=$ (full or partial) oscillatory decay plus incoherent decay

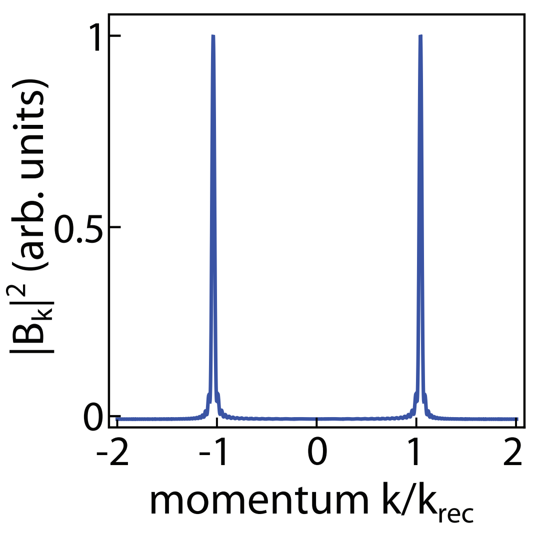
1D effective Rabi-frequency $\tilde{\Omega} = \left(\frac{\pi\Omega}{8\omega_0}\right)^{1/3}\Omega$

Non-Markovian (incomplete oscillatory) decay for Δ < Ω, especially Δ < 0



Markovian-like (exponential) decay for Δ > Ω

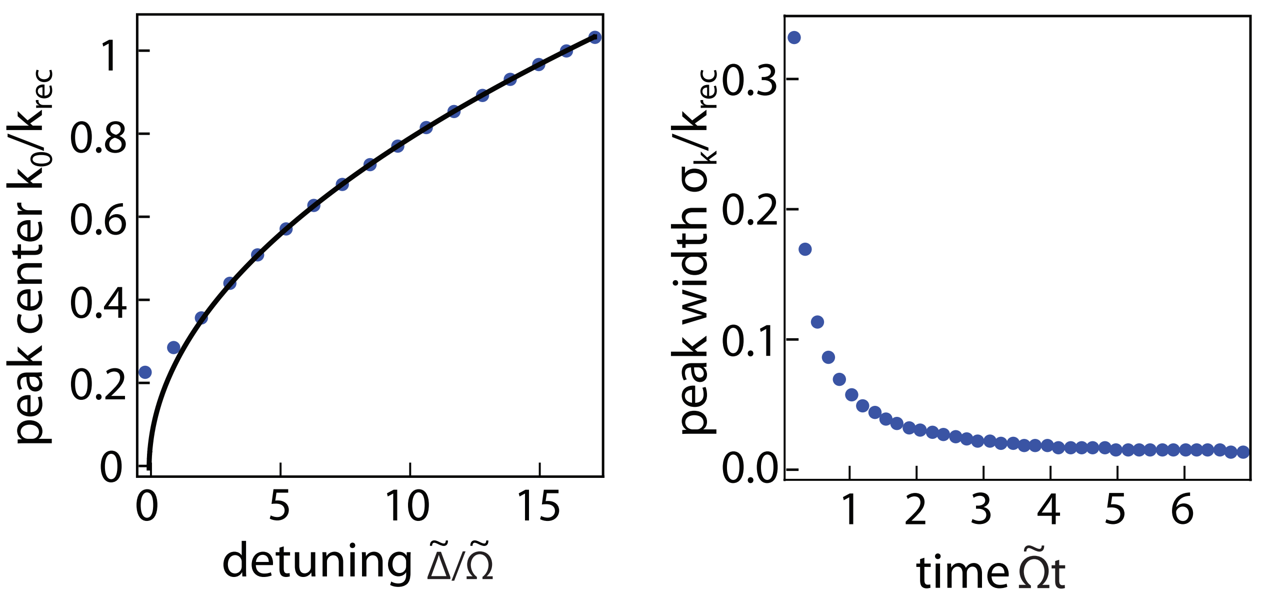
Emitted Matter Waves



- Radiated matter waves symmetric in momentum
- Evaluate numerically

$$B_k(t) = i\frac{\Omega}{2}\gamma_k\int_0^t e^{i\Delta_k t'}A(t')dt'$$

- Energy conservation determines peak centers
- Wave packets narrow in time: start Fourier limited and tend to approximately decay rate (when in Markov limit)

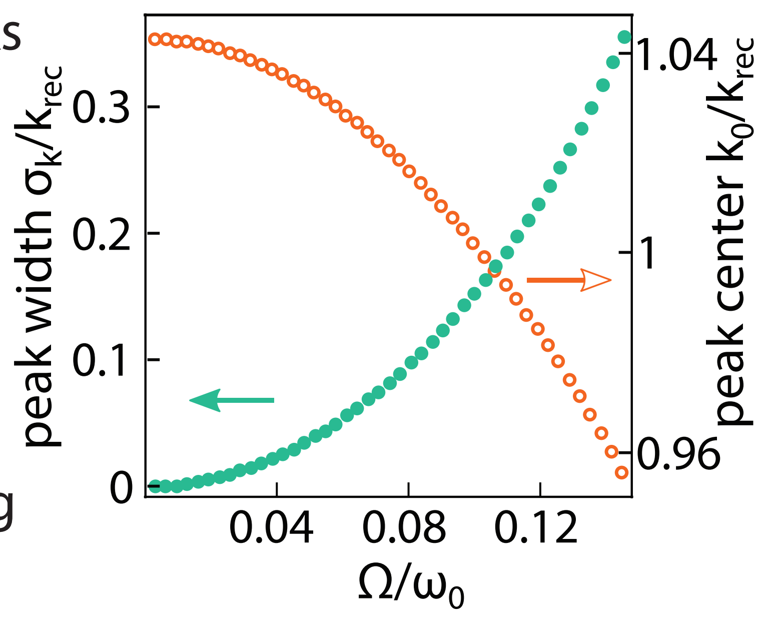


Markovian Limit

- Weisskopf-Wigner for light predicts decay with assocoated frequency shift (Lamb shift)
- Similar prediction in Markov regime of our model! (Lamb-like shift present also in non-Markovian limit)

$$\Gamma + i\delta_L \qquad \Gamma = \delta_L\sqrt{\frac{\pi\omega_0}{2\Delta}}\exp(-2\Delta/\omega_0) \qquad \delta_L = \frac{\Omega^2}{\omega_0}$$

- Shift decreases detuning pulls radiated peaks closer together for observable signature
- Peak width is power broadened for increasing coupling strength



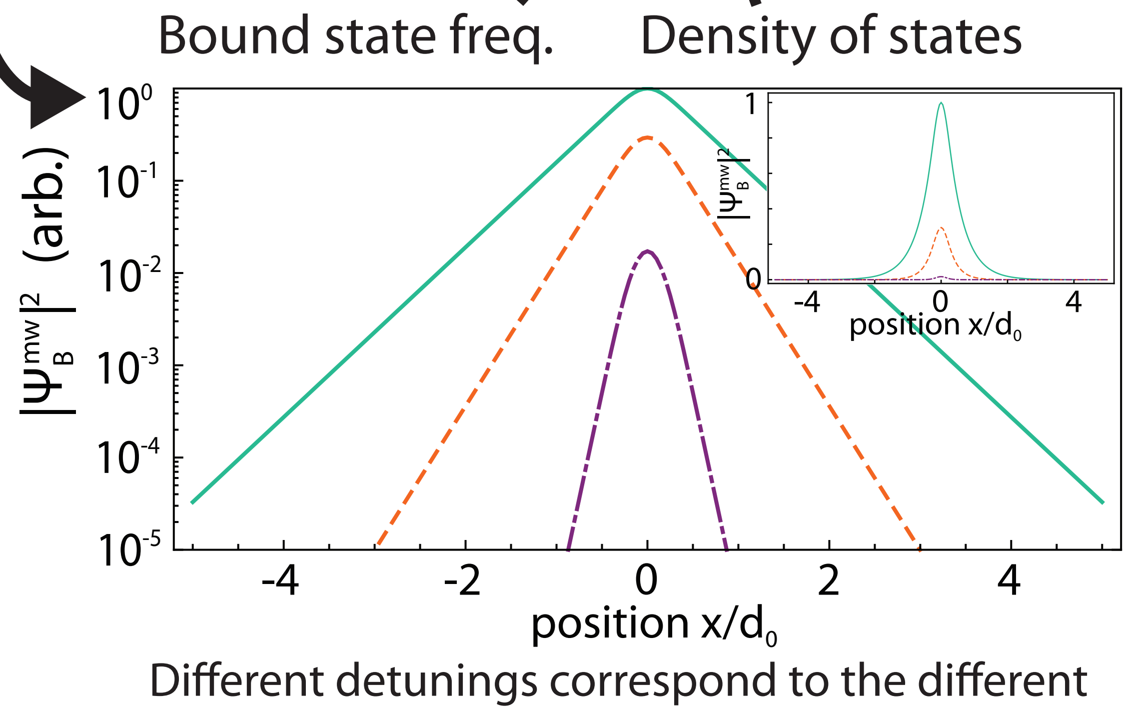
Bound State

- When coupling frequency is below the state continuum edge, form an evanescent bound matter wave state!

Coupling

$$\psi_B^{mw}(x) = c_B^{1/2}\int_0^\infty \frac{g^*(\omega)}{\omega-\omega_B}\rho(\omega)\langle x|0_a,1_\omega\rangle d\omega$$

Bound state freq. Density of states

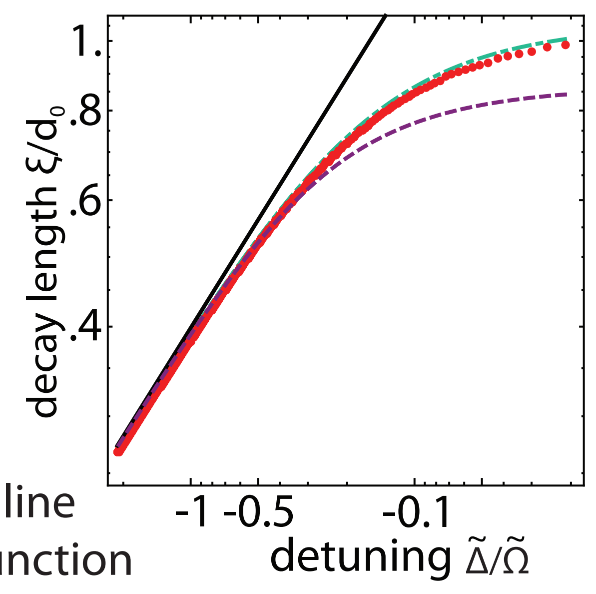


Different detunings correspond to the different curves in this plot

- Bound state exponentially localized on scale ξ

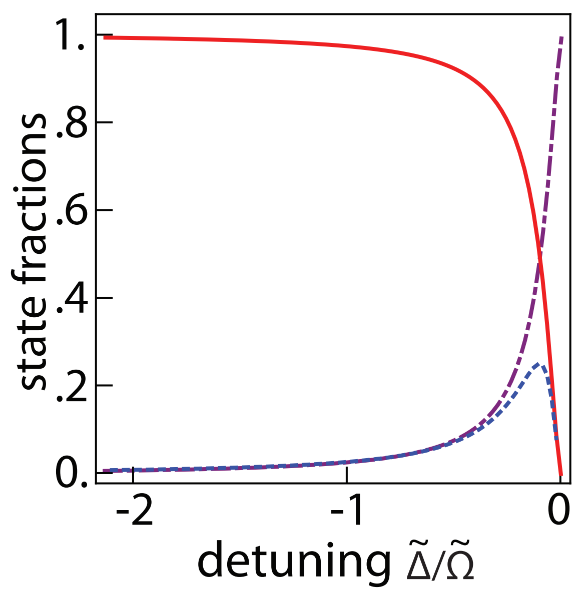
$$\xi \approx a_{ho}\sqrt{\frac{\omega_0}{2|\tilde{\Delta}|}}$$

- Many-body extension: realize long-range hopping



Different models for ξ.
Naive model is solid black line
Red dots are fits to wavefunction

- Bound state also has contribution from excited atomic emitter (red) in addition to evanescent matter waves (blue)



* This work considers a single atom in one tightly confining optical lattice well and ignore its neighbors

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