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Optics Rotation:

Levitron

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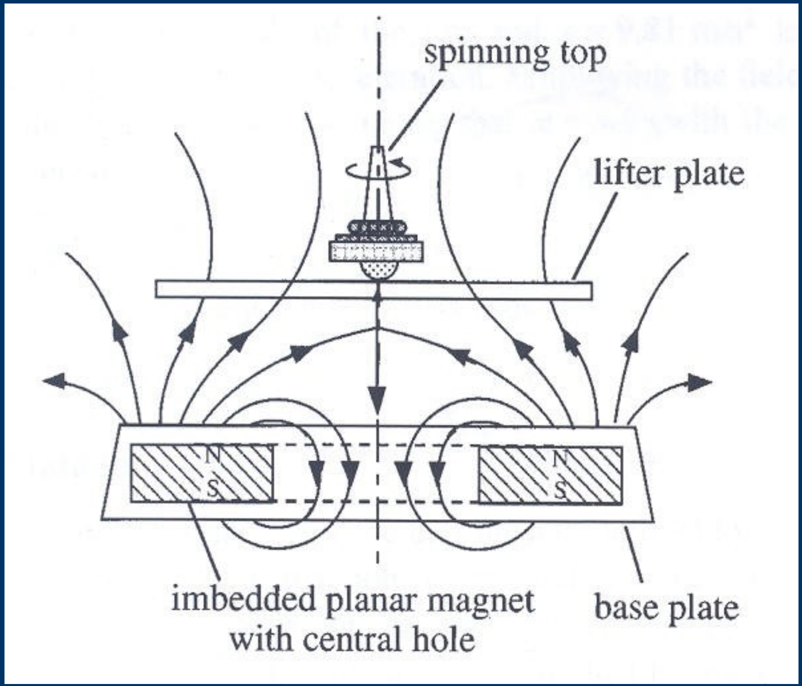
Levitron - Overview

- What is the Levitron?
- Theory
  - Basic principles
  - An simple model
- Experiment
  - Magnetic field
  - Levitation
  - Results of measurement
  - Wobbling
- Summary

Levitron – What is the Levitron?

- Stable magnetic levitation of a spinning top
  - Three parts:
    - Top: a nonmagnetic spindle inserted in a permanent magnet of the form of an annulus
    - Base: a tripod, which consists of plastic with an embedded ceramic magnet
    - Lifter plate



(1) – *Fig.1*

Levitron – Theory: Basic principles

- Equilibrium between magnetic and gravitational force:

$$\vec{\mu} \cdot \vec{\nabla} \, \vec{B} = M \, g \, \hat{z}$$

$$\textit{with magnetic moment } \vec{\mu} \quad \textit{magnetic field } \vec{B}$$

$$\textit{mass } M \quad g = 9.81 \, m / s^2$$

- The top has to be stable to radial as well as to axial displacements --> stable equilibrium

Levitron – Theory: An simple model

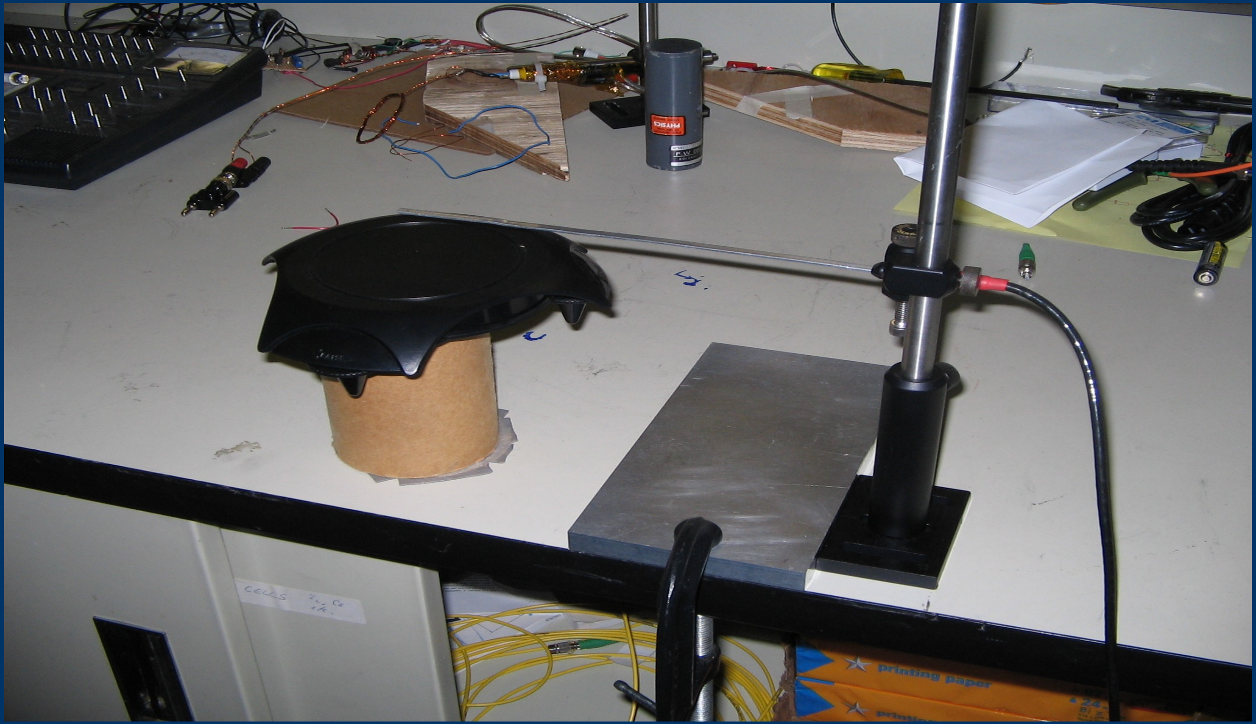
- The assumption (i), that the magnetic moment m is the whole time parallel to the z-axis due to a rapidly spinning top doesn't lead to stability. => upper limit for rotation rate
- The assumption (ii), that the top is always parallel to the magnetic field B of the base leads to a good result.
- That means: $\vec{m} = m \frac{\vec{B}}{B}$ Definition: $\alpha_n(z_0) = \frac{1}{n!} \left(\frac{1}{B_z} \frac{\partial^n B_z}{\partial z^n} \right)_{\rho=0, z=z_0}$

  $\textit{with the axial component of the magnetic field } B_z$
- Conditions for stable equlibrium:

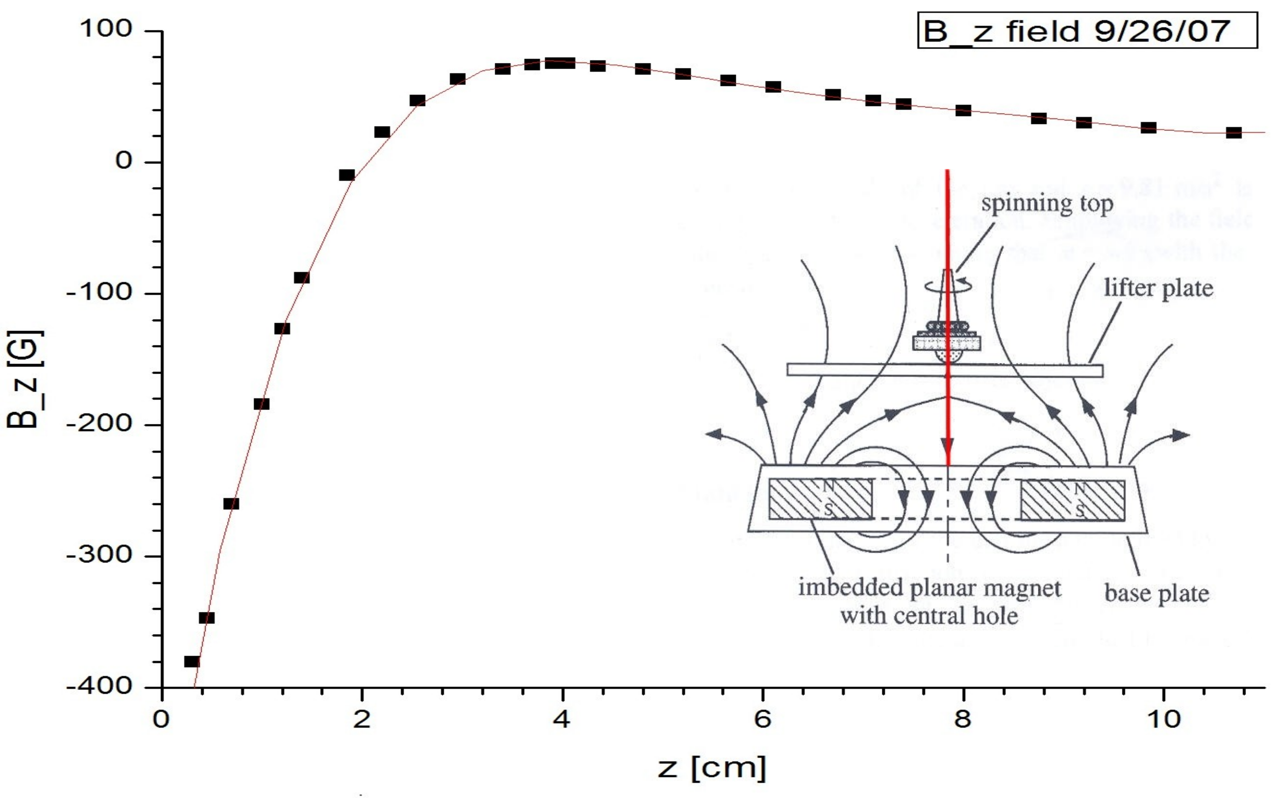
$$(i)\, m < 0 \quad (ii)\, \alpha_1 < 0 \quad (iii)\, \alpha_2 > 0 \quad (iv)\, \alpha_1^2 - 4\alpha_2 > 0$$

Levitron – Experiment: Magnetic field

- Measurement of the axial component of the magnetic field B_z as a function of the height z at the symmetry axis of the base using a Hall-effect gaussmeter probe.

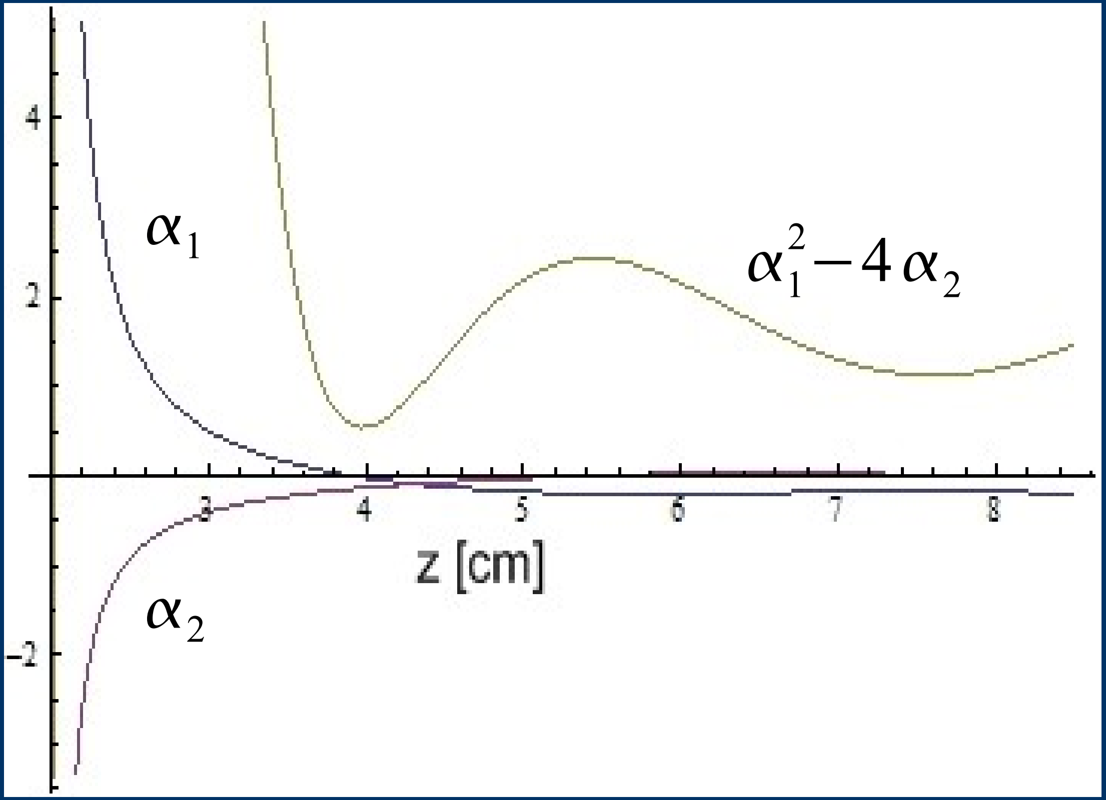


Levitron – Experiment: Magnetic field



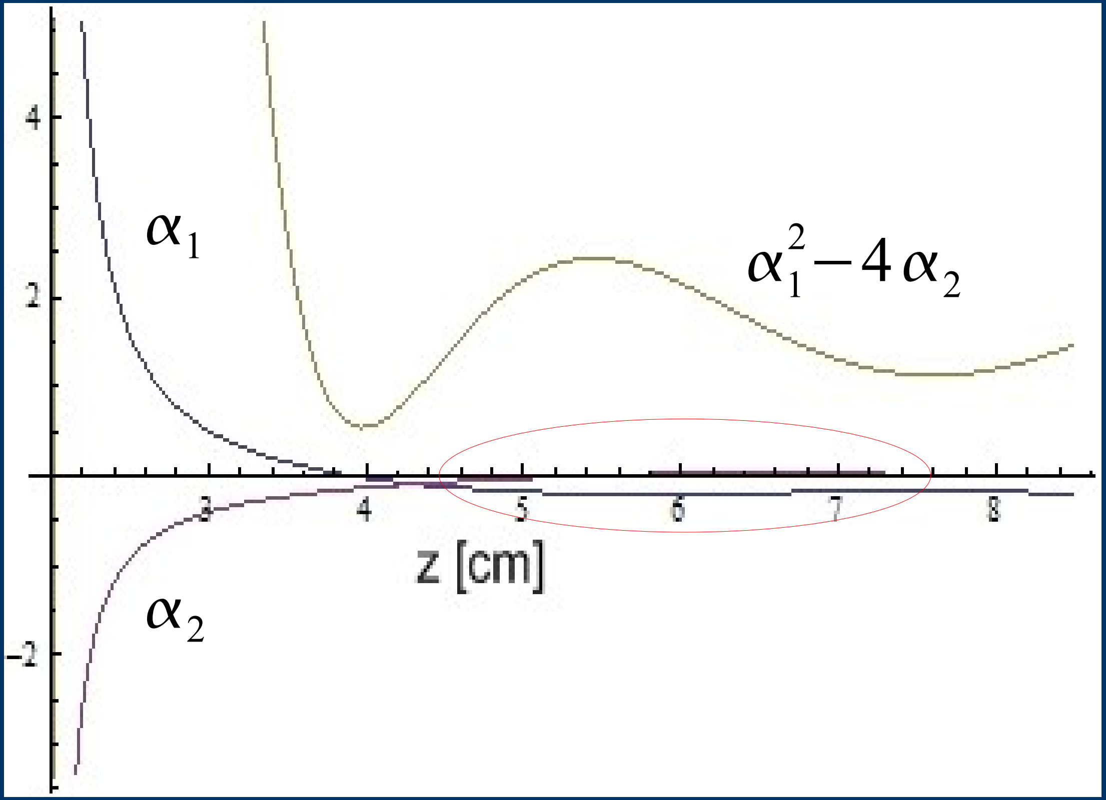
- Fitting of the data to get a fifth-order polynomial expression for axial magnetic field B_z.

Levitron – Experiment: Magnetic field



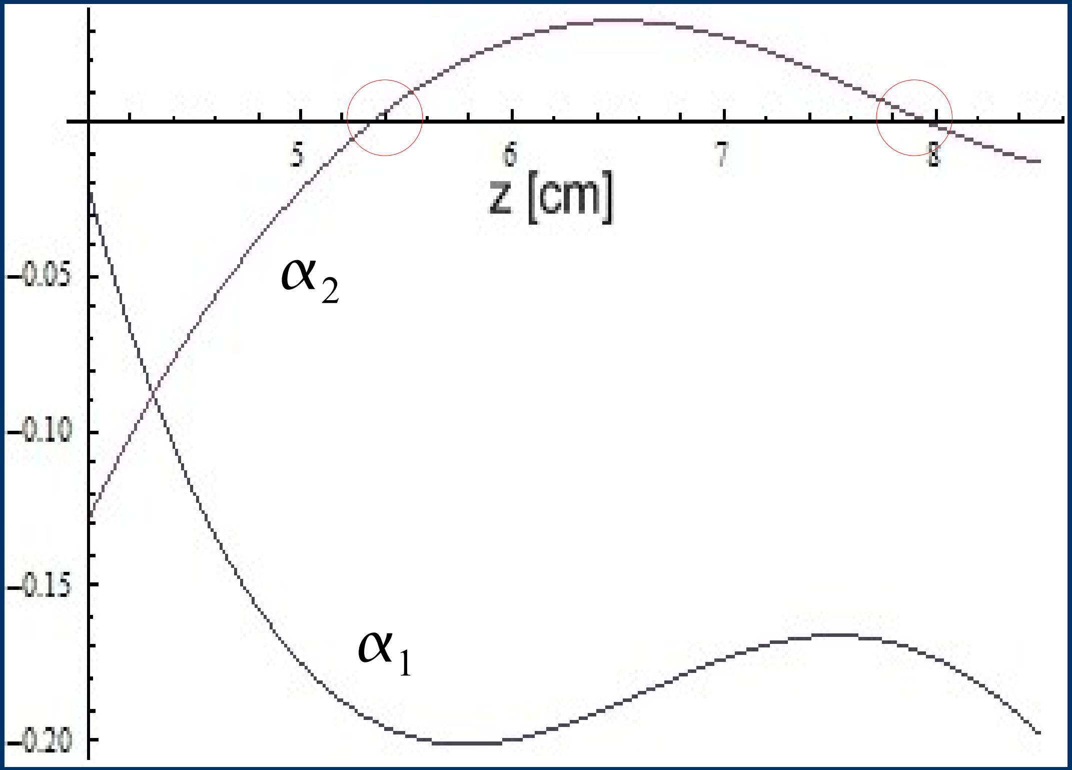
$$\alpha_n(z_0) = \frac{1}{n!}\left(\frac{1}{B_z}\frac{\partial^n B_z}{\partial z^n}\right)_{\rho=0,\, z=z_0} \qquad (ii)\, \alpha_1 < 0 \quad (iii)\, \alpha_2 > 0 \quad (iv)\, \alpha_1^2 - 4\alpha_2 > 0$$

Levitron – Experiment: Magnetic field



$$\alpha_n(z_0)=\frac{1}{n!}\left(\frac{1}{B_z}\frac{\partial^n B_z}{\partial z^n}\right)_{\rho=0,\,z=z_0} \qquad (ii)\,\alpha_1<0 \quad (iii)\,\alpha_2>0 \quad (iv)\,\alpha_1^2-4\alpha_2>0$$

Levitron – Experiment: Magnetic field



- All three conditions are fulfilled for:

$$5.34\text{cm} < x < 7.96\text{cm}$$

Levitron – Levitation

Levitron – Levitation

Levitron – Experiment: Results

- Results of the measurement:

Weight [g]	Height [cm]	del_z B_z [10^-4 T/m]	M. moment mu [Am^2]
19.25	6.9	-8.42	224.01
19.35	6.8	-8.72	217.67
19.45	6.65	-9.19	207.65
19.55	6.45	-10.7	194.66
19.65	6.2	-12.22	180.19
19.75	5.65	-12.38	158.59
19.85	5.55	-12.38	157.32
19.95	5.5	-12.44	157.35

$$\mu \approx \frac{M g}{\partial_z B_z}$$

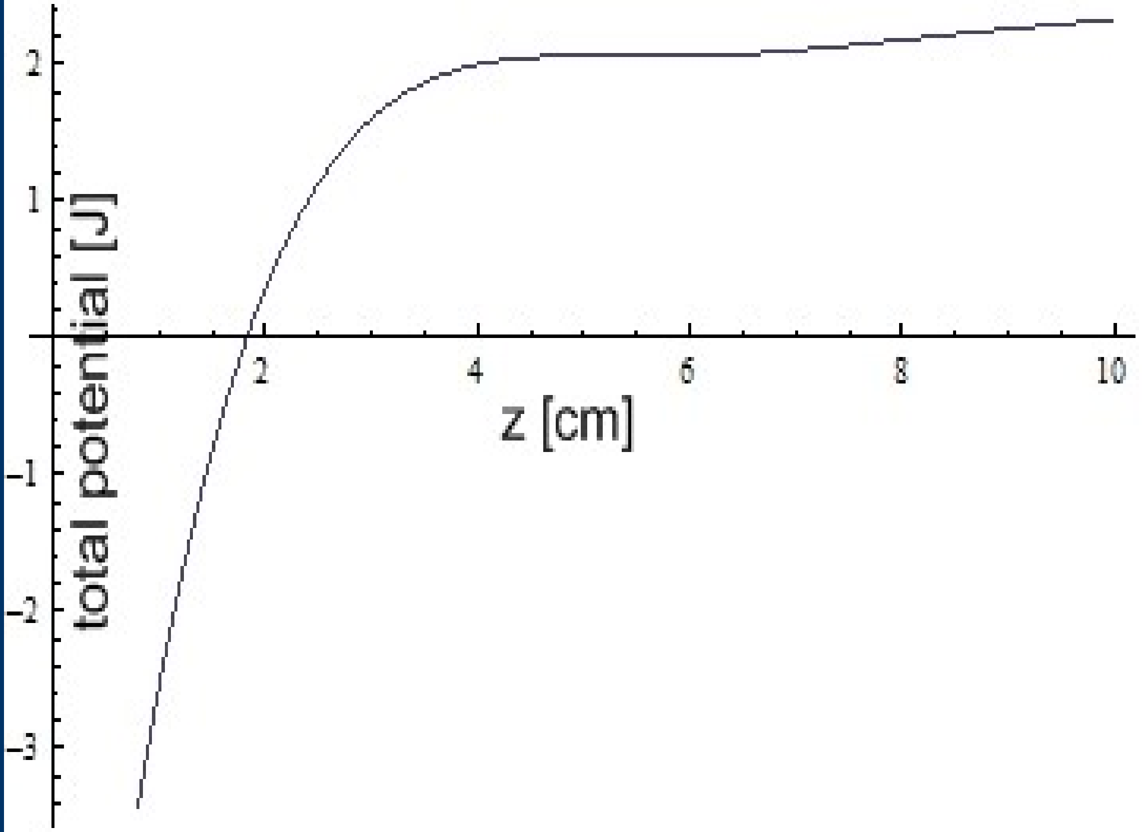
- prediction:

$$5.34\text{cm} < x < 7.96\text{cm}$$

- Magnetic moment from a measurement of the force with a spring balance: $\mu = 155.2\, Am^2$

Levitron – Experiment: Results

- Total potential $V(z) = \mu \cdot B_z(z) - M g z$ for M = 19.75g:



Levitron – Experiment: Results

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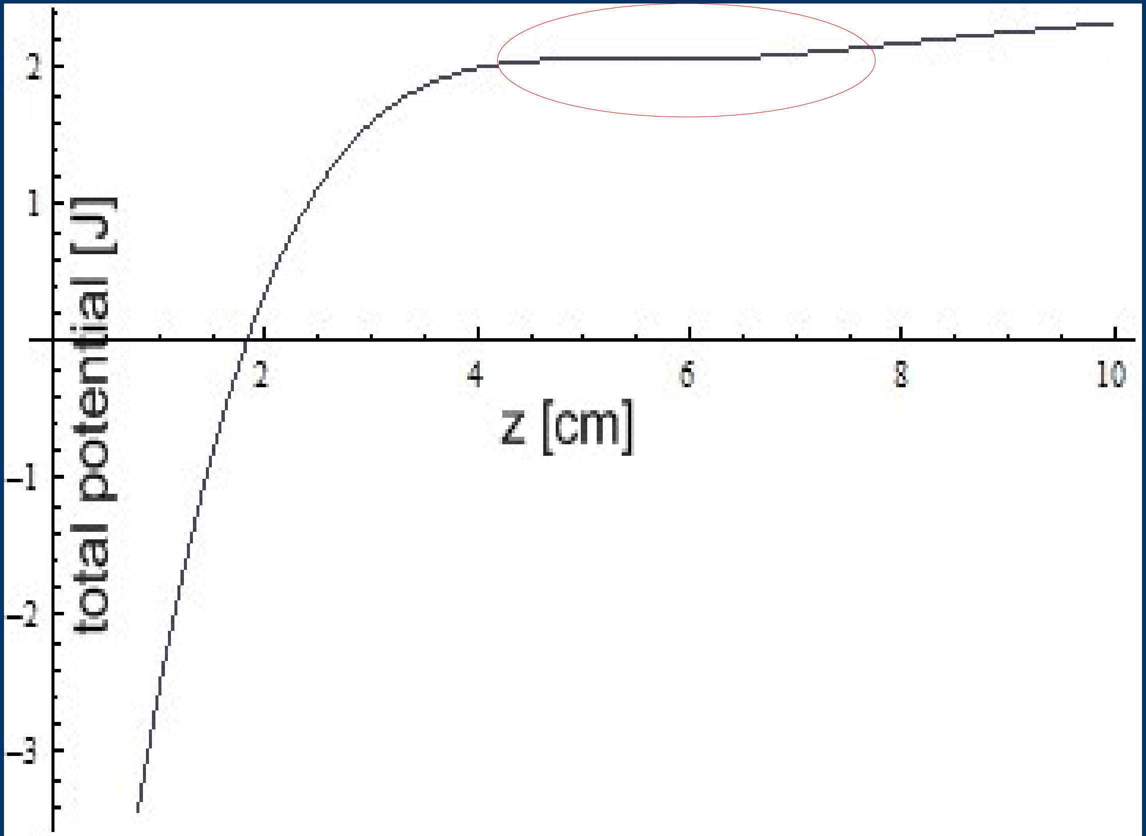
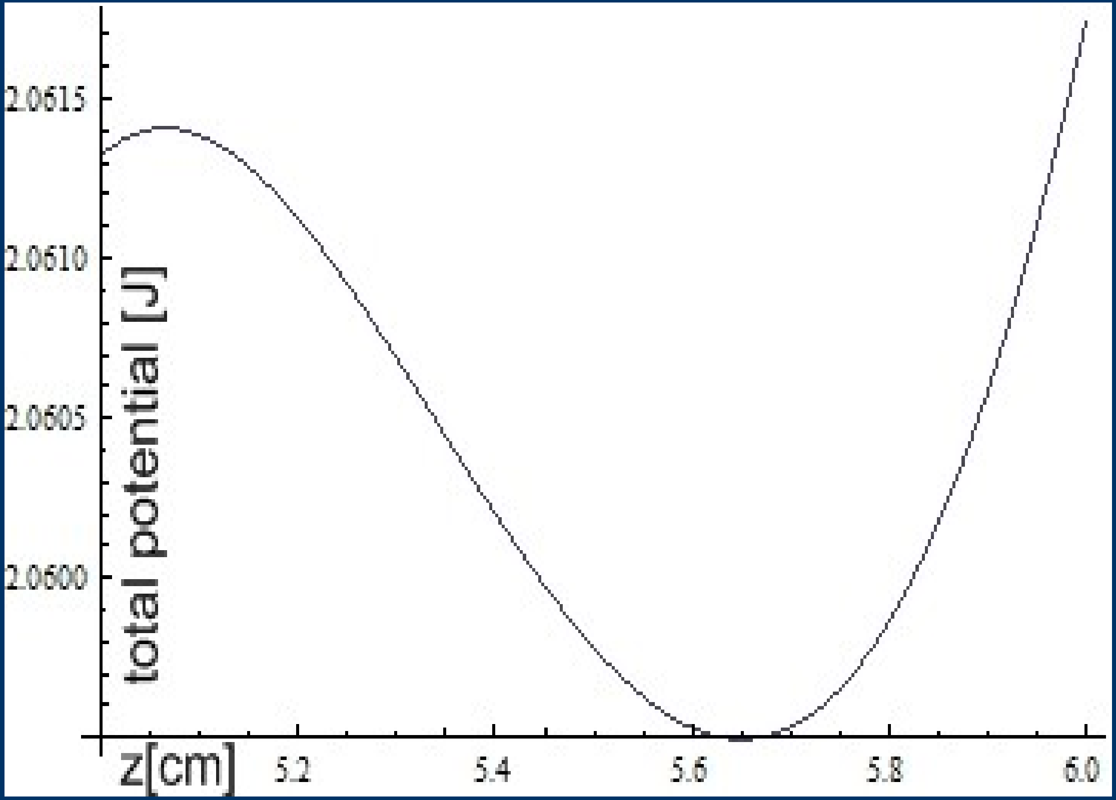


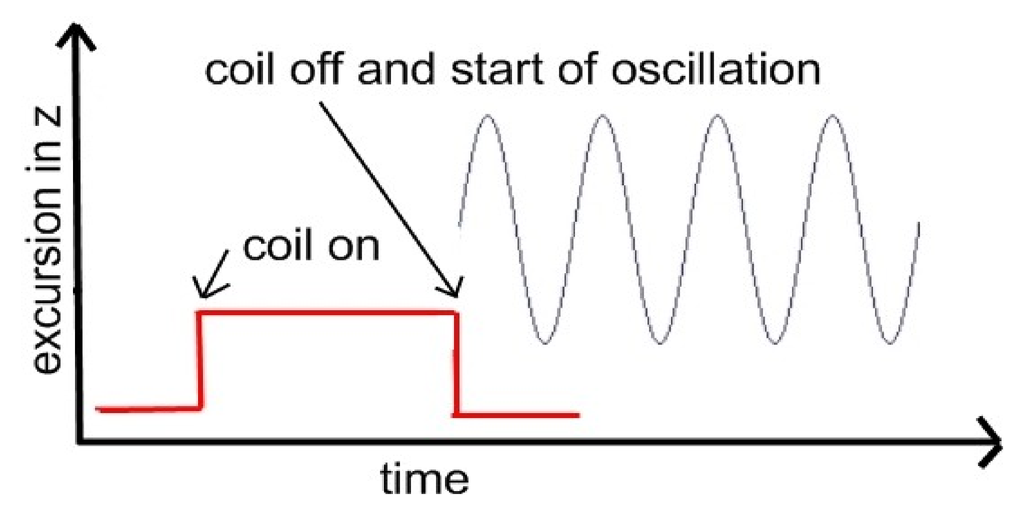
Levitron – Experiment: Results

- The mininum is at z = 5.65cm (the height is measured and the magnetic moment $\mu = 158.59\ Am^2$ is estimated):



Levitron – Experiment: Wobbling

- Perturbation of the stable levitation of the top with a magnetic field of a coil.
- The coil is fixed at the bottom of the base.
- The wire of the coil is for a short time connected to a battery --> oscillation of the top around the minimum



Levitron – Experiment: Wobbling

- Approximation of the minimum of the total potential to the potential of a harmonic oscillator

$$V(z) = \mu \cdot B_z(z) - M g z$$

- Taylor expansion:

$$V(z) = V(z_0) + V'(z_0)(z - z_0) + \frac{1}{2} V''(z_0)(z - z_0)^2 + \ldots$$

$$\Rightarrow V(z) = \frac{1}{2} V''(z_0)(z - z_0)^2 + \ldots$$

Levitron – Experiment: Wobbling

- Consideration as harmonic oscillator:

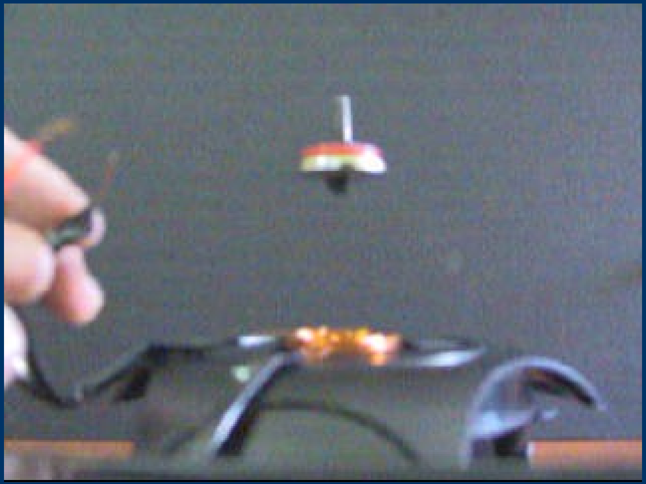
$$V(z) = \frac{1}{2} M \omega^2 (z - z_0)^2 \Rightarrow \nu = \frac{1}{2\pi} \sqrt{\frac{V''(z_0)}{M}}$$

- Example:
  - Weight of top M = 20.95g
  - Height of levitation: 5.8cm

$$\Rightarrow \mu \approx \frac{M g}{\partial_z B_z} = -172{,}84\, Am^2$$

- We get for the “trap frequency“: $\nu = 0.23\text{Hz}$

Levitron – Experiment: Wobbling



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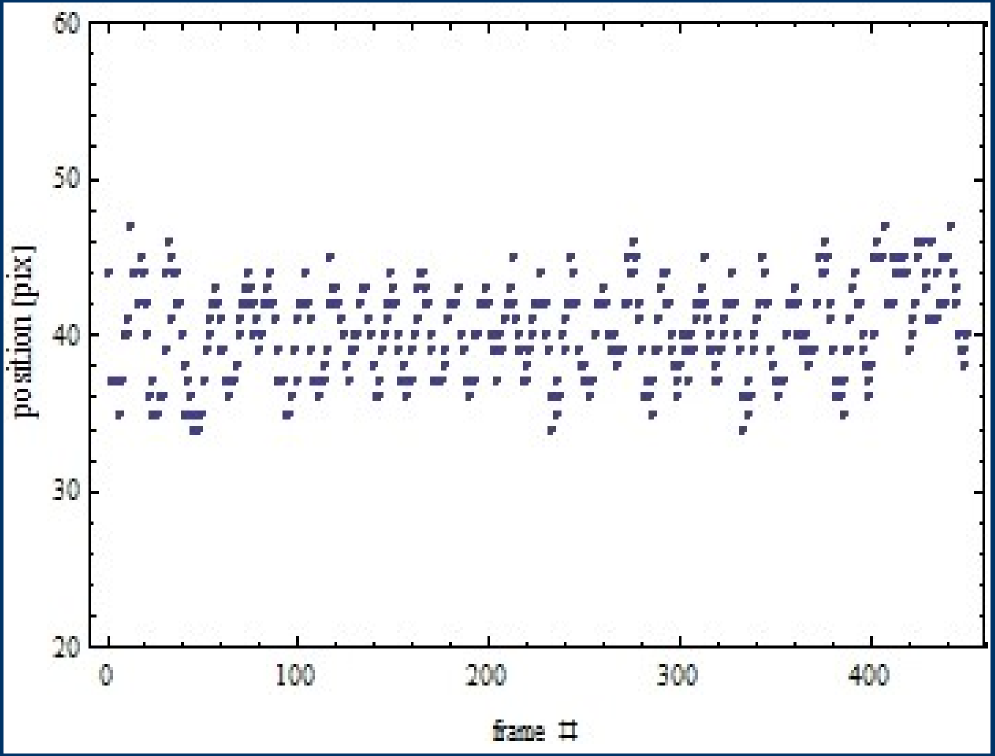

- 15 frames are 1 second

- There is already a oscillation without any perturbation through the coil caused by the airflow and the first push at the lifting.
- The “trap frequency“ is significant smaller than the already existing oscillation frequency.
- Superposition of both frequencies

Levitron – Summary

- Direct connection between weight of the top and the height of levitation
- There is only a small range where stable levitation is possible
- There is an upper and a lower limit of the rotation rate of the top for stable levitation
- The spinning is the dynamical element, it prevents the top from overturning
- Connection with traps for microscopic particles

Levitron – The end

Thank you for your attention!

Levitron – Sources

- (1) T. B. Jones, Masao Washizu, Roger F. Gans: “Simple theory for the Levitron“; J. Appl. Phys. 82 (2), 15 July 1997, p. 883-888
- (2) Michael V. Berry: “The Levitron: an adiabatic trap for spins“; Mathematical, Physical and Engineering Sciences, Vol. 452, No. 1948, 8 May 1996, p. 1207-1220
- (3) Roger F. Gans, Thomas B. Jones, Masao Washizu: “Dynamics of the Levitron“; J. Phys. D: Appl. Phys. 31, 1998, p. 671-679

Levitron – Sources

- (4) Michael V. Berry: “Frequently Asked Questions About the Levitron“; http://www.lauralee.com/physics.htm, 23 October 2007
- Acknowledgment: Dominik A. Schneble, Joe Feliciano