Optics Rotation:

Levitron

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Levitron - Overview

• **What is the Levitron?**

• **Theory**
  – Basic principles
  – An simple model

• **Experiment**
  – Magnetic field
  – Levitation
  – Results of measurement
  – Wobbling

• **Summary**
Levitron – What is the Levitron?

- Stable magnetic levitation of a spinning top
  - Three parts:
    - Top: a nonmagnetic spindle inserted in a permanent magnet of the form of an annulus
    - Base: a tripod, which consists of plastic with an embedded ceramic magnet
    - Lifter plate

(1) – Fig. 1
Levitron – Theory: Basic principles

• Equilibrium between magnetic and gravitational force:

\[ \vec{\mu} \cdot \vec{\nabla} \vec{B} = M \, g \, \hat{z} \]

*with magnetic moment* \( \vec{\mu} \)  *magnetic field* \( \vec{B} \)

*mass* \( M \)  *gravity* \( g = 9.81 \, m/s^2 \)

• The top has to be stable to radial as well as to axial displacements --> **stable equilibrium**
Levitron – Theory: An simple model

- The **assumption (i)**, that the magnetic moment \( m \) is the whole time parallel to the \( z \)-axis due to a rapidly spinning top doesn't lead to stability. \( \Rightarrow \) upper limit for rotation rate

- The **assumption (ii)**, that the top is always parallel to the magnetic field \( B \) of the base leads to a good result.

That means:

\[
\tilde{m} = m \frac{\vec{B}}{B} \quad \textbf{Definition:} \quad \alpha_n(z_0) = \frac{1}{n!} \left( \frac{1}{B_z} \frac{\partial^n B_z}{\partial z^n} \right)_{\rho=0, z=z_0} \quad \text{with the axial component of the magnetic field } B_z
\]

- **Conditions for stable equilibrium:**
  
  \((i) m < 0 \quad (ii) \alpha_1 < 0 \quad (iii) \alpha_2 > 0 \quad (iv) \alpha_1^2 - 4 \alpha_2 > 0\)
Levitron – Experiment: Magnetic field

- Measurement of the axial component of the magnetic field $B_z$ as a function of the height $z$ at the symmetry axis of the base using a Hall-effect gaussmeter probe.
Levitron – Experiment: Magnetic field

- Fitting of the data to get a fifth-order polynomial expression for axial magnetic field $B_z$. 
Levitron – Experiment: Magnetic field

\[ \alpha_1(z_0) = \frac{1}{n!} \left( \frac{1}{B_z} \frac{\partial^n B_z}{\partial z^n} \right)_{\rho=0, z=z_0} \]

(ii) \( \alpha_1 < 0 \)  (iii) \( \alpha_2 > 0 \)  (iv) \( \alpha_1^2 - 4\alpha_2 > 0 \)
Levitron – Experiment: Magnetic field

\[ \alpha_n(z_0) = \frac{1}{n!} \left( \frac{1}{B_z} \frac{\partial^n B_z}{\partial z^n} \right) \quad \rho = 0, z = z_0 \]

(ii) \( \alpha_1 < 0 \)  \quad (iii) \( \alpha_2 > 0 \)  \quad (iv) \( \alpha_1^2 - 4 \alpha_2 > 0 \)
Levitron – Experiment: Magnetic field

- All three conditions are fulfilled for:
  \[ 5.34\text{cm} < x < 7.96\text{cm} \]
Levitron – Levitation
Levitron – Levitation
Levitron – Experiment: Results

- Results of the measurement:

<table>
<thead>
<tr>
<th>Weight [g]</th>
<th>Height [cm]</th>
<th>$\Delta z \ B_z \ [10^{-4} \text{T/m}]$</th>
<th>M. moment $\mu$ [Am$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.25</td>
<td>6.9</td>
<td>-8.42</td>
<td>224.01</td>
</tr>
<tr>
<td>19.35</td>
<td>6.8</td>
<td>-8.72</td>
<td>217.67</td>
</tr>
<tr>
<td>19.45</td>
<td>6.65</td>
<td>-9.19</td>
<td>207.65</td>
</tr>
<tr>
<td>19.55</td>
<td>6.45</td>
<td>-10.7</td>
<td>194.66</td>
</tr>
<tr>
<td>19.65</td>
<td>6.2</td>
<td>-12.22</td>
<td>180.19</td>
</tr>
<tr>
<td>19.75</td>
<td>5.65</td>
<td>-12.38</td>
<td>158.59</td>
</tr>
<tr>
<td>19.85</td>
<td>5.55</td>
<td>-12.38</td>
<td>157.32</td>
</tr>
<tr>
<td>19.95</td>
<td>5.5</td>
<td>-12.44</td>
<td>157.35</td>
</tr>
</tbody>
</table>

- Magnetic moment from a measurement of the force with a spring balance: $\mu = 155.2 \text{ Am}^2$

- Prediction:

$\mu \approx \frac{M \ g}{\partial_z B_z}$

$5.34 \text{cm} < x < 7.96 \text{cm}$
Levitron – Experiment: Results

- Total potential $V(z) = \mu \cdot B_z(z) - M \cdot g \cdot z$ for $M = 19.75$g:
Levitron – Experiment: Results

- Total potential \( V(z) = \mu \cdot B_z(z) - M \cdot g \cdot z \) for \( M = 19.75 \text{g} \):
Levitron – Experiment: Results

- The minimum is at $z = 5.65\text{cm}$ (the height is measured and the magnetic moment $\mu = 158.59 \text{Am}^2$ is estimated):
Levitron – Experiment: Wobbling

- Perturbation of the stable levitation of the top with a magnetic field of a coil.
- The coil is fixed at the bottom of the base.
- The wire of the coil is for a short time connected to a battery --> oscillation of the top around the minimum
Levitron – Experiment: Wobbling

• Approximation of the minimum of the total potential to the potential of a harmonic oscillator

\[ V(z) = \mu \cdot B_z(z) - M g z \]

• Taylor expansion:

\[ V(z) = V(z_0) + V'(z_0)(z - z_0) + \frac{1}{2} V''(z_0)(z - z_0)^2 + \ldots \]

\[ \Rightarrow V(z) = \frac{1}{2} V''(z_0)(z - z_0)^2 + \ldots \]
Levitron – Experiment: Wobbling

- Consideration as harmonic oscillator:

\[ V(z) = \frac{1}{2} M \omega^2 (z - z_0)^2 \Rightarrow \nu = \frac{1}{2\pi} \sqrt{\frac{V''(z_0)}{M}} \]

- Example:
  - Weight of top \( M = 20.95\)g
  - Height of levitation: 5.8cm

\[ \Rightarrow \mu \approx \frac{M g}{\partial_z B_z} = -172,84 \, Am^2 \]

- We get for the “trap frequency“: \( \nu = 0.23\)Hz
Levitron – Experiment: Wobbling
Levitron – Experiment: Wobbling
Levitron – Experiment: Wobbling

- There is already an oscillation without any perturbation through the coil caused by the airflow and the first push at the lifting.
- The “trap frequency” is significantly smaller than the already existing oscillation frequency.
- Superposition of both frequencies

- 15 frames are 1 second
Levitron – Summary

- Direct connection between weight of the top and the height of levitation
- There is only a small range where stable levitation is possible
- There is an upper and a lower limit of the rotation rate of the top for stable levitation
- The spinning is the dynamical element, it prevents the top from overturning
- Connection with traps for microscopic particles
Levitron – The end

Thank you for your attention!
Levitron – Sources

Levitron – Sources

• (4) Michael V. Berry: “Frequently Asked Questions About the Levitron“;
• Acknowledgment: Dominik A. Schneble, Joe Feliciano