

PHY 300 lab information

The lab notebook should document what you did on an experiment, how you did the analysis, and what your results and conclusions were. Here are some comments based on the Michelson Interferometer lab.

First of all, the only way to do the lab properly is to be prepared! Read the lab writeup beforehand. Think through what the most important parameters of the measurement are; often your error will be dominated by one parameter and if you can improve its measurements you can greatly improve your results.

Your lab notebook should record the date of the experiment, and the people on the experimental team. You should list the name of the lab and a 1–3 sentence description of your goals in the lab. If your apparatus has a serial number, record that. Make a simple schematic sketch (not an artistic drawing!) of your apparatus, and indicate relevant settings. Note that for the measurement of the refractive index of air in the Michelson Interferometer lab, you have the rare situation of needing to record one environmental parameter: the barometric pressure at the time of your experiment (you can probably get that off of a web site with a listing of the weather at Islip airport).

Describe the procedure of setting up your apparatus. In the case of the Michelson Interferometer, you should add any comments you find useful to the alignment procedure in the lab handout, and make sketches of how the fringe patterns change as you get the system into alignment. As you take data, make a clear table of your results, indicating both the units and the uncertainty in each measurement. If an entire column of measurements has the same uncertainty, you can simply list this for the first value and the same uncertainty is then implicitly applied to all subsequent values.

There are a variety of ways you can improve your experimental procedure. As one example, consider the act of turning a micrometer knob. When you turn it one way, you are causing a lead screw to have threads make contact in one direction and push on the part to be moved; when you are turning it the other way, the threads are making contact on an opposite side and often you are relying on a spring to push the moving part back in contact with the lead screw end. Therefore the best way to adjust a micrometer is to overshoot when unscrewing and then always make your final adjustment in the screwing-in direction, thereby eliminating backlash as a systematic error in your measurements. This is one example; if you think through how your apparatus works you are likely to come up with procedures for maximum accuracy.

It is essential that you do a quick analysis of your data during your lab session! This will help you find out if you have forgotten to determine any key parameters, or if you are somehow using erroneous units for some value, or if something has gone wildly wrong on the lab. This need not be exhasutive; for example if you have a long table of points (such as fringe number versus vacuum in the Michelson Interferometer lab when you are measuring the refractive index of air), you might simply do a calculation of the last point of data rather than a careful plot and line fit of all data, though a plot can also tell you if things went haywire. Your T.A. should not sign off on your attendance at lab until you can show that you have reasonable results.

In the aftermath of your experiment, you now want to carry out a more complete analysis of your results. Show all relevant formulae, and show your uncertainty calculations (you don't need to spell out every detail of every repetitive uncertainty calculation; showing the detail for a representative data point is OK). Note that the use of a spreadsheet program such as Microsoft

Excel can remove much of the tedium of this work, and it will also give you nice printouts of your analysis tables and graphs. Make note of things you realize about your data, such as pointing out the factors that might dominate the accuracy/systematic error and those that dominate random error in the experiment. Include any relevant plots in your writeup (if you do things like the plotting and analysis on a computer, it's perfectly fine to paste printouts into your lab notebook).

You want to end with a conclusion to your lab. In very brief sentences, describe what you learned, how your measurements worked out, how your results compare with accepted values, and any thoughts you have for how to improve your measurements were you to do the lab again.

It may be the case that your result comes out slightly different than the accepted value, even when your uncertainty is taken into account. If you are off by a factor of 2 or 100 or something large, you had better ask hard questions of yourself because you probably made a serious mistake somewhere (was a calibration knob mis-set?). (But of course this won't happen to you because you have already done a quick analysis of your results in the lab room before leaving). Beyond that, it is important that you report the result you obtained in all honesty even if it's not what you expected. There are at least two reasons for doing so: "fudging" or altering results to get what you expect rather than what you measured constitutes academic fraud, and of course there are numerous examples in the history of science where those who made more careful repeats of experiments obtained significantly different values that stood the test of time.

Above all, realize that this is a lab notebook and not an essay. You don't need to repeat all that is in your lab handout; it is assumed that anyone who repeats the experiment will have access to the same handout. However, what you do want is make it clear in the end what you determined from your experiment and what your numerical results were.

1 Error analysis

As you probably have learned by now, physics gives particular meanings to certain terms associated with error analysis:

Error or uncertainty refers to the best estimate of the quantitative range within which one can trust his or her results. The error can be calculated even if there is no "official" accepted value for what is being measured. To John Q. Public, "error" refers to how badly he screwed up, or how far off his answer is from the "official" value.

Systematic error refers to an error that is consistent from measurement to measurement. For example, if you always round down to the nearest tic mark on a meter stick when measuring length, you will make a systematic error of measuring a slightly shorter length.

Random error refers to an error which fluctuates in an unpredictable fashion from measurement to measurement. For example, if you were asked to determine the number of births per day at Stony Brook University Hospital, you would likely get a slightly different number each day you inquired even though the total number of births per year might remain fairly steady from year to year.

Accuracy refers to the degree to which our value is correct within uncertainty. This is largely a matter of having the correct calibration of all of our reference measurements. That is, if someone gave us a miscalibrated meter stick that was shorter than the official length of a meter, we might measure the length of a table with great precision (lots of decimal places) but poor accuracy (what we think is a meter is not really a meter).

Precision can be thought of as the number of meaningful digits to a measurement. A measurement of a length as being 1.03424 meters is more precise than a measurement of 1.03 meters; however, if the measurement was made with an incorrectly calibrated meterstick, the measurement will have high precision but low accuracy!

Reference standard is the thing that you measure against. For example, if we're measuring the length of various tables, we might use a meter stick for the measurement. The meter stick is then our reference standard for the measurements of the lengths of the tables, and if our reference standard is inaccurate, then all measurements made using it will also be inaccurate no matter how precise they are. The international system of units, or SI units (for *Le Syst'eme International d'Unités*), tries as much as possible to use reference standards based on natural phenomena that anyone can replicate in a properly-equipped lab (mass is the glaring exception to this principle; mass is defined relative to a particular [platinum-iridium cylinder](#) sitting in Paris). The [base units](#) are a choice of seven well-defined units which by convention are regarded as dimensionally independent: the metre, the kilogram, the second, the ampere, the kelvin, the mole, and the candela. As an example, the SI unit definition of the second is as follows:

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.

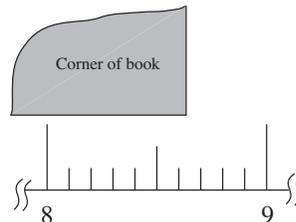
The name of the game in our lab activities is to provide estimates of the error of all of our measurements.

In this course, we will use only SI units. The U.S. is one of the only countries left on the planet that does *not* use SI units in everyday transactions; it may be that Liberia is the only other country that shares this characteristic. Even within our own country, SI units are becoming more and more prevalent within industry because of international trade. If you dare use inches, or pounds, or gallons in this course, you will be publicly humiliated and 0.01 points will be taken off of your grade.

SI units grew out of the metric system that was created during the French Revolution, and originally the unit of length was derived from a measurement of the diameter of the earth. There's lots of good information out there on SI units; good places to start are the *Bureau International des Poids et Mesures* or [BIPM web site](#), [Wikipedia on SI](#), and the National Institute of Standards and Technology or [NIST web site](#).

2 Measuring a length

Let's say that you wanted to measure the width of your textbook, and when you held it up with a ruler aligned to one edge you saw the following at the other edge:



Everyone can agree that the number is between 8.6 and 8.7 centimeters or cm, but it's tough to decide whether it's 8.63 or 8.64, for example. A reasonable way to report this might be to say that it is 8.63 ± 0.03 cm, or 8.65 ± 0.05 cm. In writing these numbers, notice that we did not go beyond the precision of the error in specifying the value; that is, it makes no sense to write 8.63247 ± 0.1 cm. Also note that a number without units is meaningless in this context; if you were to just say that the length was about 8.6 it would be impossible to be certain if the length was 8.6 cm or 8.6 yards, for example. Also, the exact measure of the length of the book is only as good as A) the calibration of your meter stick and B) how accurately you were able to line up the zero point of the meter stick with the other end of the book. Also, if the book has a very rough edge, it may be difficult to say exactly where the edge is; you should account for that in reporting the error of your measurement (if the edge were bumpy or wavy at the level of 0.2 cm, you would say that the length was 8.6 ± 0.2 cm). Generally speaking, your estimate of the error should include all factors that you think provide a reasonable estimate of how well others can trust your measurement.

3 Error propagation

Errors can propagate in measurements. Consider the charade of first-down yardage measurements in football. There's an unavoidable random error in where the on-field official marks a ball down at the end of a play. However, this position is then eyeballed by a yard marker official on the sideline, who attempts to mark the same position from some distance away. If there's a call for an on-field yardage measurement, the chain gang comes out. The crucial person is the one off-camera who eyeballs the yardage marker on the sideline and plants one end of the chain there. We then see the theater of the chain being stretched out, and the official making a great show out of showing precisely how the ball is a few inches short of a first down while the TV commentators get all excited. What a load of crap! Because there has already been a propagation of errors from two eyeball measurements of position transfer, it would be no less accurate for the on-screen official to simply eyeball the ball's position relative to the first down marker on the sideline and make a call without all of this unnecessary drama.

But let's get back to the imaginary experiment of measuring your book. Let's say that its width is x and its height is y and that we wish to calculate the area $A = x \cdot y$ of the book. If there's some error to the width x of Δx , and some error to the height y of Δy , then it's pretty obvious that there

is going to be some error in the calculated area A :

$$\begin{aligned} A &= (x \pm \Delta x)(y \pm \Delta y) = x\left(1 \pm \frac{\Delta x}{x}\right)y\left(1 \pm \frac{\Delta y}{y}\right) \\ &= xy \left(1 \pm \frac{\Delta x}{x} \pm \frac{\Delta y}{y} \pm \frac{\Delta x}{x} \frac{\Delta y}{y}\right) \simeq xy \left(1 \pm \frac{\Delta x}{x} \pm \frac{\Delta y}{y}\right) \\ &\simeq xy \pm (\Delta x + \Delta y) \end{aligned}$$

where we have assumed that $\Delta x/x$ and $\Delta y/y$ are both small so that $(\Delta x/x)(\Delta y/y)$ is really small and can be safely ignored. That is, we simply add the two fractional errors to get the net error. In fact, if the two errors are uncorrelated (that is, there's no reason to expect that an undermeasurement of x is associated with an undermeasurement of y), we should really add the two errors in a root mean square fashion:

$$\Delta A = \Delta(xy) = \sqrt{(\Delta x)^2 + (\Delta y)^2} \quad \text{for } A = x + y \quad (1)$$

Also, note that if one error is significantly larger than the other, we can get a good idea of the error by just considering the dominant term in such a root mean square or RMS sum.

It turns out that similar rules apply for division errors, addition errors, and power law errors:

$$\Delta A = \sqrt{(\Delta x)^2 + (\Delta y)^2} \quad \text{for } A = x + y \quad (2)$$

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \quad \text{for } A = x/y \quad (3)$$

$$\frac{\Delta A}{A} = \sqrt{2\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \quad \text{for } A = x^2 y \quad (4)$$

$$\frac{\Delta A}{A} = \sqrt{\frac{1}{2}\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \quad \text{for } A = x^{1/2} y \quad (5)$$

and so on. We can generate rules for other operations (like $\sin()$ and $\cos()$, for example) by using a first-order Taylor series expansion

$$f(x - x_0) \simeq f(x_0) + (x - x_0) \frac{d}{dx} f(x)|_{x=x_0} \quad (6)$$

to come up with the right rule for any operation. The assumption here is that the errors are small.

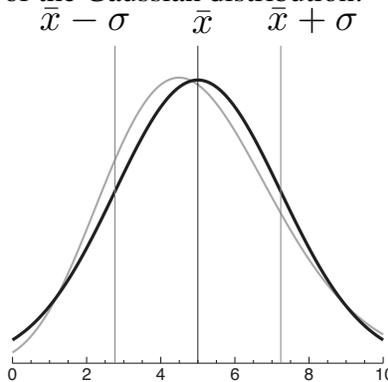
4 Random error

In some measurements, there is a random element involved. Let's say that it is your job to measure the fraction of times that a coin lands face up. You might refuse to make the measurement, saying that you know the answer: it's going to land face up exactly 50% of the time. OK, but what if you make two measurements? If you flip the coin twice, do you expect it to land face up once, and face down once, every time you flip it twice? Of course not! Since each flip of the coin is uncorrelated with the previous flip (the coin has no reason to remember how it landed last time),

there is an intrinsic measurement error which for our purposes we can approximate as being equal to the square root of the number of events, or \sqrt{N} . That is, if you flip a coin 10 times so that you would expect to have 5 heads, about two-thirds of the time you'll find that the number of heads you get is within the range $5 - \sqrt{5} \simeq 3$ and $5 + \sqrt{5} \simeq 7$, and that one third of the time you'll get fewer than 3 or more than 7 heads! In other words, you expect to get something like a Gaussian distribution of events about a mean value of \bar{N} of

$$P(N, \bar{N}) = \frac{1}{\sqrt{2\pi\bar{N}}} \exp \left[-\frac{(N - \bar{N})^2}{2\bar{N}} \right] \quad (7)$$

where $P(N, \bar{N})$ means “the probability of measuring N this particular time, in the case where the average of many measurements is \bar{N} .” This has the same form as $\exp[-(x - \bar{x})^2/2\sigma^2]$, so that $\sigma = \sqrt{\bar{N}}$ characterizes the width of the Gaussian distribution:



That is, about 2/3 of the points will fit between $\bar{x} - \sigma$ and $\bar{x} + \sigma$ on a Gaussian distribution. Now this is only approximately right:

- The Gaussian plotted above is for a continuous distribution in x , whereas we started out talking about integer numbers of events N . Plotting such events should really give a kind of a staircase distribution, because we have only integer value positions on the x axis.
- For events for which no negative value is possible, the real story is that we should use a Poisson distribution (shown in grey above) which is subtly different from a Gaussian distribution for $\bar{x} \lesssim 10$. This is discussed in more detail in Appendix ??.
- The real story is that for a Gaussian distribution one can calculate the number of points that fall outside of some multiple of σ by using the error function $\text{erf}(x)$ as described in Appendix ??.

Still, the above curve gives you an idea.

Random error can come in all kinds of measurements beyond simple flipping of coins. For example, when measuring the period of a pendulum, you have to hit a stopwatch with some human reaction time thrown in; this might have a mean (which you can deal with by anticipating the right moment for stopping the watch based on observing the event transpire, or by including it in both the start and the end time), and it might also have a variance σ^2 . It's this latter variance that usually factors into measurements you will make.

5 Curve fitting

Frequently in the laboratory you will have the situation that you perform a series of measurements of a quantity y at different values of x . This can often give you a much more accurate determination of a physical parameter than a single measurement will provide, since you are now able to see a correlation of resulting values y over a range of test values x . When you have linear relationship of $y = mx + b$, you can determine the uncertainty in the measured slope m and the intercept b . One way to do this is through a least-squares fit, which is a procedure to minimize the squared error χ^2 of

$$\chi^2 = . \quad (8)$$

Simple least square fits are usually available in spreadsheet programs (it's the `LINEST` function in Excel, for example), and more specialized programs can allow for different uncertainties of the data points and for fits of polynomials, exponentials, Gaussian, and other types of functions to the data. However, if you have no access or experience with spreadsheet programs, you can use a simple, graphical method. Plot the measured points (x, y) and mark for each point (or at least for some representative points) the errors Δx and Δy as bars that extend from the plotted point in the x and y directions. Draw the line that best describes the measured points (*i.e.*, the line that minimizes the sum of the squared distances from the line to the points to be fitted; the least-squares line). This line will give you the best value for slope m and intercept b . Next, draw the steepest and flattest straight lines that are consistent with the error bars of at least two-thirds of the data points. The difference in slope m between the flattest (m_{\min}) and steepest (m_{\max}) lines should indicate the 2σ range of the slope m , so that the 1σ error Δm in the slope will be given by $\Delta m = (m_{\max} - m_{\min})/2$.

Note that you can also handle relationships other than simple linear by appropriate transformation of the data. If you have measurements of position x versus time t under constant acceleration such that $x = (1/2)at^2$, you can plot \sqrt{x} versus t and obtain a fit of $\sqrt{x} = mt$ in which case you can make the association $a = 2m^2$.

A Comparison of Poisson and Gaussian distributions

If the average number of events obtained in many separate measurements is \bar{x} , then the probability of counting x events in one particular measurement is given by the Poisson distribution

$$P(x, \bar{x}) = \frac{\bar{x}^x}{x!} \exp(-\bar{x}). \quad (9)$$

The difficulty in using the exact expression for the Poisson distribution is that both x^x and $x!$ increase dramatically for even modest values of x . We wish to find an approximate expression for the Poisson distribution both for analytical and for numerical calculations. This is done using Stirling's approximation, which says

$$x! \simeq \sqrt{2\pi x} x^x \exp\left[-x + \frac{1}{12x} + O(x^{-2})\right]. \quad (10)$$

We can then approximate the Poisson distribution as

$$P(x, \bar{x}) \simeq \frac{1}{\sqrt{2\pi x}} \left(\frac{\bar{x}}{x}\right)^x \exp[x - \bar{x}] \exp\left[-\frac{1}{12x}\right]. \quad (11)$$

In the case where $x \gg \bar{x}$, the term $(\bar{x}/x)^x$ will be small while the term $\exp[x - \bar{x}]$ will become large. To avoid problems in numerical evaluation of the Poisson distribution, we therefore want to combine these terms as follows:

$$\begin{aligned} \ln\left\{\left(\frac{\bar{x}}{x}\right)^x \exp[x - \bar{x}]\right\} &= x \ln\left(\frac{\bar{x}}{x}\right) + x - \bar{x} \\ &= x \left[1 + \ln\left(\frac{\bar{x}}{x}\right)\right] - \bar{x}. \end{aligned} \quad (12)$$

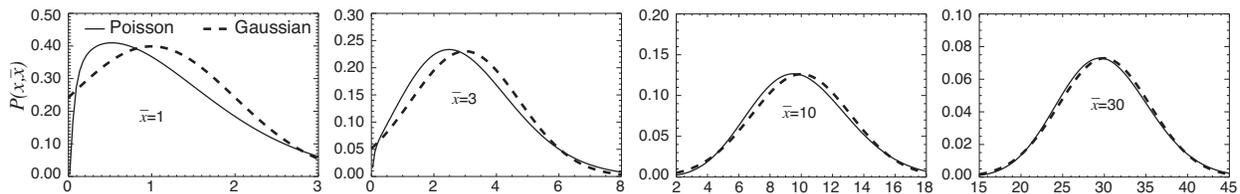
We then write the Poisson distribution as

$$P(x, \bar{x}) \simeq \frac{1}{\sqrt{2\pi x}} \exp\left\{x \left[1 + \ln\left(\frac{\bar{x}}{x}\right)\right] - \bar{x}\right\} \exp\left[-\frac{1}{12x}\right]. \quad (13)$$

We now have an expression that is more easily evaluated numerically, but it is still too cumbersome to use in simple analytical calculations. For those cases, we can approximate the Poisson distribution with a Gaussian distribution:

$$P(x, \bar{x}) = \frac{1}{\sqrt{2\pi\bar{x}}} \exp\left[-\frac{(x - \bar{x})^2}{2\bar{x}}\right] \quad (14)$$

with a truncation to $P = 0$ for $x < 0$ so as to exclude the incorrect non-zero values of P for negative event numbers x due to the long tails of the Gaussian distribution. Here's a comparison between Poisson and Gaussian distributions for small values of \bar{x} :



As you can see, the Gaussian distribution with the limit $P(x < 0) = 0$ is a quite accurate and usable approximation for $\bar{x} \gtrsim 10$.

B The error function $\text{erf}(x)$

The statement that two-thirds of the measurements fall within a range of 1σ of the mean is only approximate. A precise statement is provided by the error function $\text{erf}(x)$ of

$$\text{erf}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x \exp[-t^2] dt \quad (15)$$

which has particular values of

σ	x	$\text{erf}(x)$
1	$1/\sqrt{2}$	0.6827
2	$2/\sqrt{2}$	0.9545
3	$3/\sqrt{2}$	0.9973

and values over a larger range as follows:

